Thoughts on CSBs

The present spreadsheet tool available treats the wheels as point loads on a beam which is simply supported at the fulcrum points. It calculates the deflection of the wheel loads at their point of application.

The basic underlying assumption is that the wheel loads can be calculated before the calculation of the deflections and that the wheel loads remain unaffected by the deflection. There is also a further underlying assumption that the CSB beam undergoes deflections which can be regarded as "small" in relation to its span. The latter assumption is quite standard in the analysis of structures but it is questionable whether it is actually correct with CSBs. However the additional complexity it would bring by not assuming it would be large and therefore I won't consider that any further.

The problem remains however regarding the assumption that the wheel loads are not related to the deflection. That cannot be the case.

I would prefer to look at the problem as it actually is, that is with the wheels acting as the supports and the fulcrums applying the load. Figure 1 illustrates.

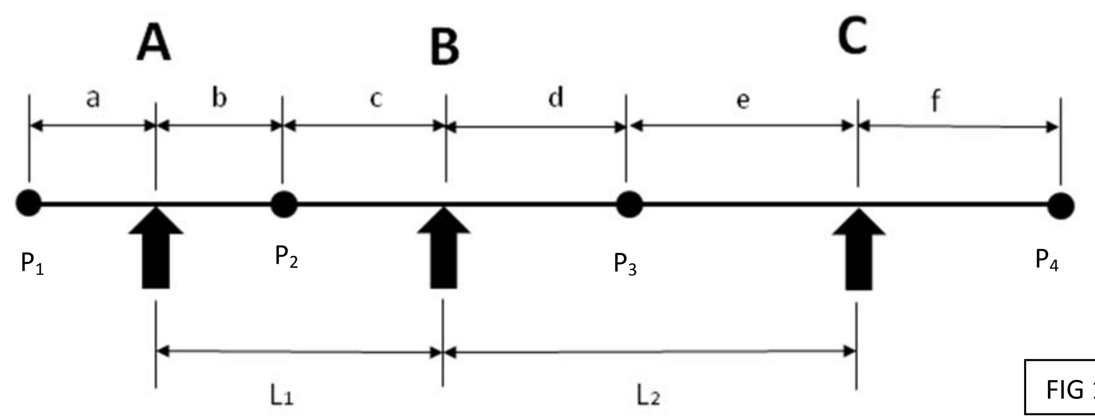


FIG 1

Where the arrows are the wheels and the dots are the fulcrum points.

To analysis this we would apply point loads at the fulcrum points and undertake some relatively easy and well known calculations to derive the bending moments at the points of support and beneath the loads. With knowledge of the Bending Moments the deflections at those points can then be calculated. This is basically what the spreadsheet does now but upside down if you understand me.

The problem arises with the point loads. What are they and how do we calculate them?

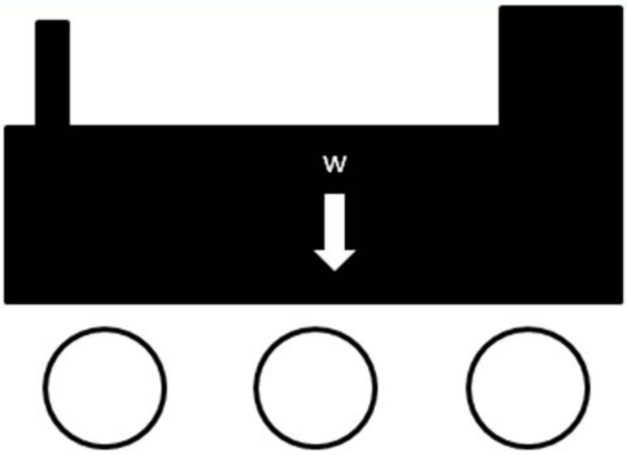


FIG 2

The figure 2 illustrates the problem. The naive assumption would be that each wheel carries 1/3 of W. That cannot be correct. A more so realistic assumption would be that the outer wheels carry ¼W and the middle wheel carries 1/2W. That assumption can be illustrated by.

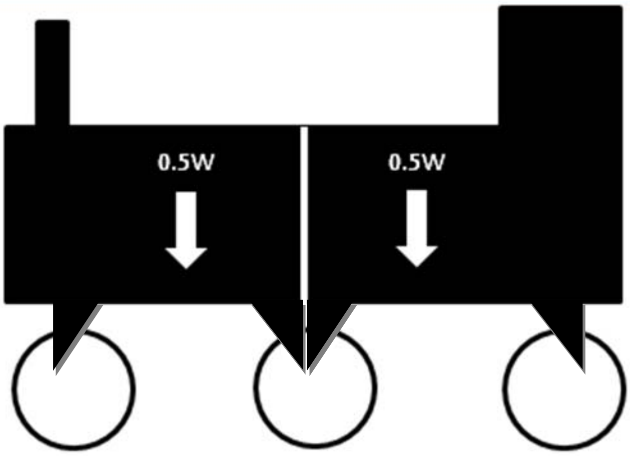


FIG 3

In other words this assumes that the front bit of the load is independent of the rear part of the load. As you can imagine if the centre wheel goes up and down the load does not change.

Now look back at Figure 2. Here we have the weight of the locomotive held in a rigid structure and if you let the centre wheel drop or in the extreme removed it is easy to see that the load will increase to the outer wheels and eventually they will each carry 1/2W. It is for this reason that the underlying assumption of the spreadsheet is not correct. Unfortunately the problem does materially affect the deflection calculation and cannot be ignored. Further, unfortunately, the problem is complex and would require a very sophisticated spreadsheet to undertake the calculation if not a full blown structural analysis programme.

That said I consider that if certain rules are adopted in the design of CSBs then these problems can be minimised to the point of acceptance. These are:

- The problem is analysed as it is i.e. the fulcrum points apply the load.

- That the deflection at each fulcrum point is equal or lies on a straight line drawn through the fulcrum points.
- The problem of the centre of gravity of the load not being coexistent with the midway point between the outer two fulcrum points can also be dealt with.

So the problem would be represented by figure 4:

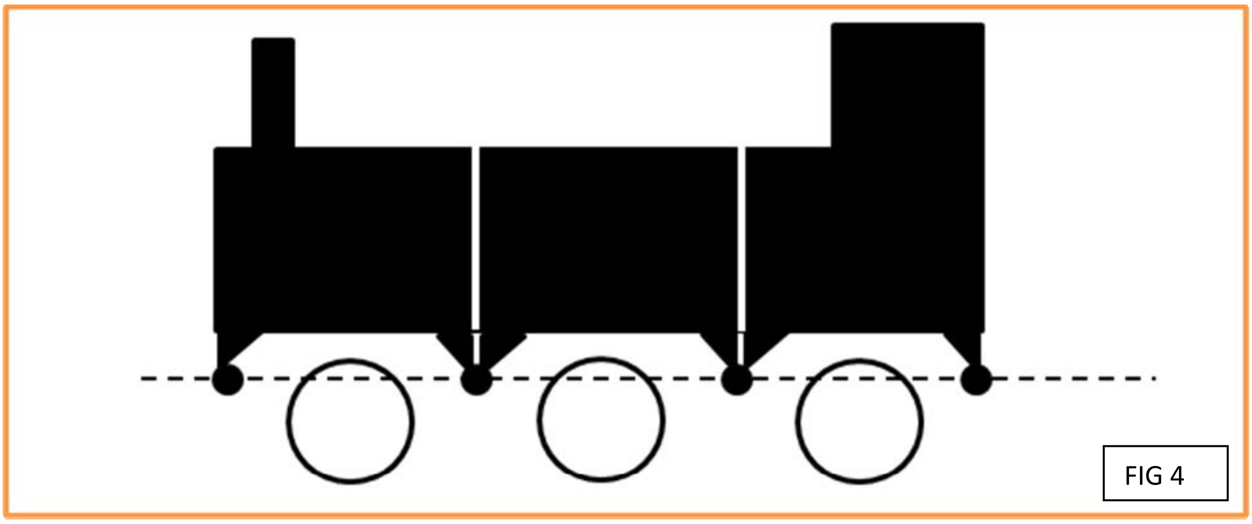


FIG 4

I belive that this assumption is valid PROVIDING that the deflection of each fulcrum point is the same or lies on a straight line drawn through the fulcrums. In other words the gaps between the illustrated loads do not open or close up. The reason for this is that the fulcrom points are held in a straight line by the frames and therfore that must be the outcome of the analysis.

The problem of the non-central load can be dealt with by measuring the load on each outer fuldrum (probably more practical to measure at each buffer beam) and then derive the load diagram as illustrated in figure 5

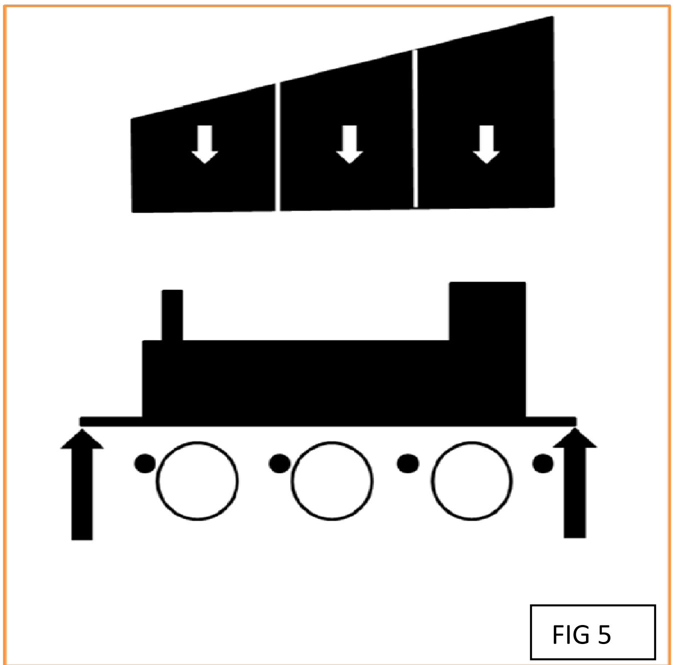


FIG 5

The load diagram can be split and the loads applied to the fulcrum points.

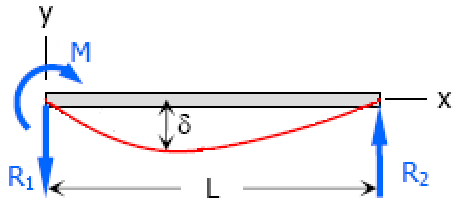
Refering to Figure 1 the bending moments at A, B and C can now be calculated.

M_A is trivial being given by the load on that particular fulcrum times the length ‘a’ and similarly for $M_{C.}$ using dimension ‘f’.

The central moment (M_B)at B can be derived from Claperoyan’s equation, which (for point loads)is:

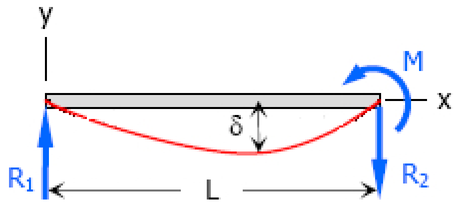
$$M_A W_1 + 2.M_B(W_1+W_2) + M_C.W_2 = P_2/L.b.(W_1^2-b^2)/L+P_3/L.e.(W_2^2-e^2)$$

And the deflections from superimposition of the standard equations so:



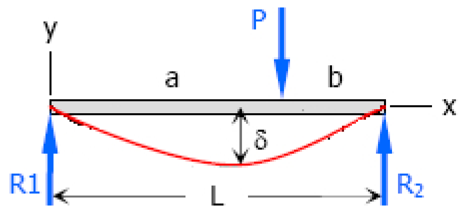
Deflection Equation (y is positive downward)

$$EI\,y = \frac{Mx}{6L}(L-x)(2L-x)$$



Deflection Equation (y is positive downward)

$$EI\,y = \frac{MLx}{6}\left(1-\frac{x^2}{L^2}\right)$$



Deflection Equation (y is positive downward)

$$EI\,y = \frac{Pbx}{6L}(L^2-x^2-b^2) \text{ for } 0 < x < a$$

$$EI\,y = \frac{Pb}{6L}\left[\frac{L}{b}(x-a)^3+(L^2-b^2)x-x^3\right] \text{ for } a < x < L$$

You will notice that:

- The stiffness of the CSB does not affect the distribution of the moments (providing that the stiffness is constant throughout its length – which is the case). That is EI does not appear in Claperoyan’s equation (providing EI is constant).
- The reactions at the wheels need not be calculated (unless you want to that is).
- That the stiffness of the wire need only be applied at the end of the calculation in order to derive the actual deflection and therefore only one calculation per wheel base/fulcrum combination is required. The effect of different wire diameters can be examined by use of simple calculation rather than a reanalysis.

The above is easily incorporated into a spreadsheet and with a little more work can be extended to 4 or more axles.

However I would emphasise that it relies on the fulcrum deflections being equal or lying on a straight line. What cannot be analysed is the situation where a “softer” central spring is desired. That requires the use of a full blown structural analysis package.